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Acoustic Radiation from Axisymmetric Ducts: A Comparison of Theory and Experiment

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A special cylindrically symmetric integral representation of the exterior solutions of the Helmholtz equation is used to calculate the free-field acoustic radiation patterns around two finite axisymmetric bodies, a straight pipe and a jet engine inlet. The radiation patterns around these bodies are then measured experimentally, with the free field being approximated through the use of an anechoic chamber. The inlet tested has a hard wall while the straight pipe is tested with both a hard and a lined wall. The computed theoretical and the measured experimental acoustic radiation patterns are found to be in good agreement. A discussion of possible sources of error, both theoretical and experimental, is included.

Introduction

ONE of the major problems facing the aircraft industry today is to reduce the noise radiated to the ground from aircraft engines without sacrificing any of the overall efficiency of the aircraft. A major source of engine noise is the compressor or fan noise which is radiated out of the jet engine through its inlet section. Research efforts directed toward reducing the noise from these sources have included the reduction of the sound level in the jet engine inlet section by the addition of sound-absorbing materials (i.e., acoustic liners). The weight and volume of these acoustic liners are of prime concern to the aircraft industry as they are directly related to the overall efficiency of the aircraft; thus, efficient acoustic liner designs are sought. In the past, optimum liner designs have been found by extensive full-scale testing of engines with various liner configurations which is a very costly and time-consuming process. Most of this testing could be eliminated by the development and use of efficient, accurate analytical procedures for the prediction of the sound field radiated from lined jet engine inlets. The development of such a method, based upon a special integral representation of the radiation solutions of the Helmholtz equation, has been discussed at length by the authors of this paper in earlier publications.[1-4] In the present study the applicability of this integral solution technique is investigated by comparing its predictions with the results obtained from an experimental study.

In previous studies,[1-4] the authors presented an integral formulation of the Helmholtz equation. The method was used to predict sound fields radiated from complicated geometries with complex (mixed) boundary conditions. The solutions generated were found to be in excellent agreement with "exact" solutions calculated by employing the method of separation of variables. A detailed development of the basic integral equations and numerical solution procedures used along with many comparisons with exact solutions for various bodies, both three-dimensional and axisymmetric, can be found in Refs. 1-5. In this connection it should be pointed out that this theoretical technique and numerical solution procedure have been found to be both accurate and computationally efficient when compared to other methods.

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In this paper a special cylindrically symmetric integral representation of the exterior solutions of the Helmholtz equation[3,4] is employed to theoretically calculate the free-field acoustic radiation patterns surrounding two finite axisymmetric bodies, namely, a straight duct and a jet engine inlet configuration. The inlet used in these studies is the QCSEE (quiet, clean, short-haul experimental engine) inlet of Ref. 6.

Theoretical Method

In this study a so-called "integral technique" is employed to calculate the sound radiated from two axisymmetric bodies. The particular method is unique in that it is applicable at all nondimensional wave numbers ka (where k is the wave number and a is an appropriate body dimension), it contains no tangential derivatives on the surface of the body and it contains no singular kernels which cannot be handled numerically by straightforward means. The details of the derivation of the basic integral equations are presented in Refs. 1, 2, and 5, and the details of the derivation of the particular cylindrically symmetric form of the equations used to generate the theoretical points in this paper are given in Ref. 3. It is noted here that although the integral equations for the acoustic potential used are cylindrically symmetric and therefore do not restrict the form of the boundary conditions (as all tangential modes may be solved for separately) they do restrict the bodies to be axisymmetric. However, only axisymmetric acoustic radiation is considered in this particular study.

The only difference between the analysis presented in Ref. 3 and the analysis used in this study is that a homogeneous Robin boundary condition is employed here (i.e., the acoustic admittance y is simply taken as the ratio of the outward normal acoustic velocity at the surface of the body to the acoustic potential). It should also be noted here that the value of the complex coupling constant in the equations in Ref. 3 is taken in this study to be the pure imaginary number i/k.

The numerical methods used to solve these equations are also presented in Ref. 3 and therefore are not repeated here. The only differences between the numerical methods used in Ref. 3 and in this paper are that a two-point rectangular rule integration formula is used along the body instead of the 4-point Gauss-Legendre formula and a 96-point instead of a 20-point Gauss-Legendre integration formula is used in the circumferential direction.

The Test Bodies

In the present study, two axisymmetric bodies, a straight duct and a jet engine inlet, are considered. The theoretical and experimental configurations do not compare exactly, as an accurate description of the "back side" of the experimental bodies entails the use of too many theoretical points on the bodies. Therefore, different external terminations are given to the theoretical models in the interest of conserving computing time and computer storage space. In this connection it has been found through theoretical studies that the exact form of the rear termination of the body has little effect on the sound field radiated in the forward half plane. Since only the sound field in the forward half plane is of interest, this approximation is not considered to be a major source of error in this study.

The first axisymmetric body considered is a straight duct. The geometry of the theoretical model is presented in Fig. 1 and the experimental geometry is shown in Fig. 2. Figure 1 also illustrates the hemispherical termination used to simplify the theoretical calculations. The L/a (length-to-radius ratio) of the duct equals 2.110.

In the experimental model the driver is placed at the throat of a nozzle section, which assures that the sound waves are as plane as possible at the driver plane where a 6.4 mm (¼ in.) condensor microphone is located to provide a reference sound pressure level (SPL) in decibels. All of the tests were conducted at nondimensional wave numbers ka below the $1T$ (first tangential) mode of the duct (i.e., $ka \leq 1.84$), thus assuring a plane wave at the driver plane. This has been verified experimentally by sweeping the reference microphone radially across the duct and it has been found that there is less than 1 dB in amplitude and 5 deg in phase variation between the wall and the center of the duct. In the theoretical model the existence of plane wave excitation at the driver plane is assumed.

The straight duct was tested in two configurations: with a hard wall and a lined wall. The lined-wall configuration consisted of 180 Helmholtz resonators (9 axial rows by 20 circumferential rows). A sketch of one of the resonators is shown in Fig. 3. For the hard-walled tests the small holes inside the duct were simply covered with tape. At it turns out this was not an altogether satisfactory procedure. This will be discussed more fully in the ensuing sections.

The second axisymmetric body tested was a model of an actual engine inlet with an L/a of 2.0. The exact mathematical form of the curves that make up the inlet contours can be found in Ref. 6. The theoretical model shown in Fig. 4 has a hemispherical termination similar to the one used for the straight duct. In the experimental model (see Fig. 5) the centerbody was held in place by four small wing cross-sectional struts set at 90 deg angles. The microphone measurement for the amplitude and phase at the driver plane was made half way between two of these struts. Again, the small condensor microphone was swept across the driver plane radially to check for the presence of plane wave excitation which was found to exist within the same limits as for the straight duct (i.e., less than a 1 dB change in amplitude and 5 deg in phase across the driver plane).

Experimental Setup and Procedures

A plan view of the anechoic chamber with a typical test setup is presented in Fig. 6. The anechoic chamber has interior dimensions of 3.1×4.0×2.0 m (10×13×6½ ft) high. The acoustic insulation used in the chamber is fiberglass which is approximately 0.6 m (2 ft) thick. The ducts tested (i.e., the straight duct and the inlet) and the microphones used for the field measurements were 0.9 m (3 ft) off the floor. The field measurements were taken 101.6 cm (40 in.) from the center of the duct exit plane on a circular arc at 11¼ deg increments from the centerline of the duct. Brüel & Kjaer 12.7 mm (½ in.) microphones were used in these tests for the field measurements.

Free-field measurements were made in the chamber to check for reflections. For the distances of interest in the chamber (i.e., up to 101.6 cm (40 in.) from a source) free-field conditions were generally approximated to within 3 dB in amplitude and 10 deg in phase.

The procedure for the tests was to take the measured SPL in decibels from the 6.4 mm (¼ in.) B & K microphone at the driver plane and input the data into the theoretical model

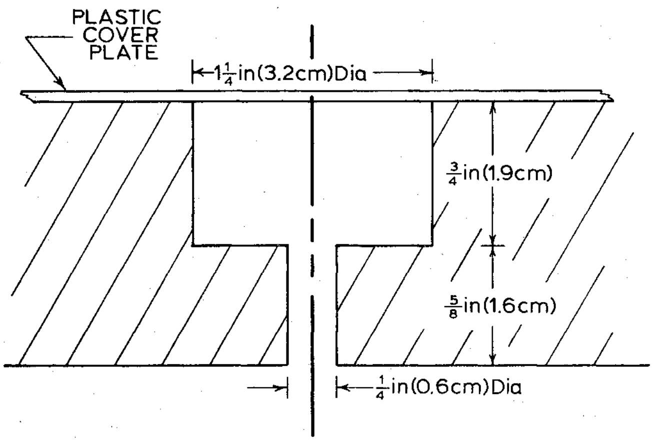

Fig. 3 Helmholtz resonator.

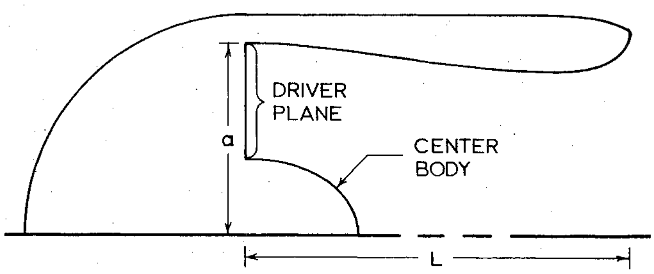

Fig. 4 Theoretical inlet model.

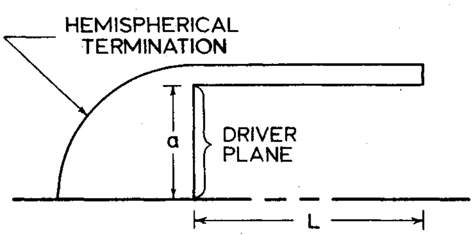

Fig. 1 Theoretical straight duct model.

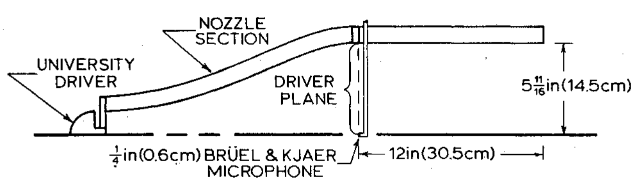

Fig. 2 Experimental straight duct test configuration.

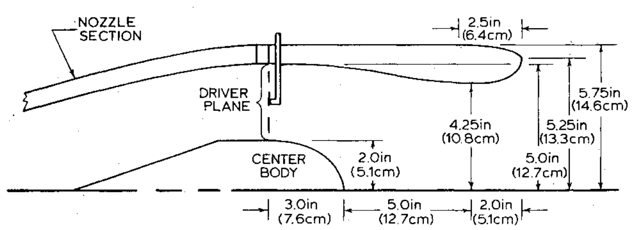

Fig. 5 Experimental inlet test configuration.

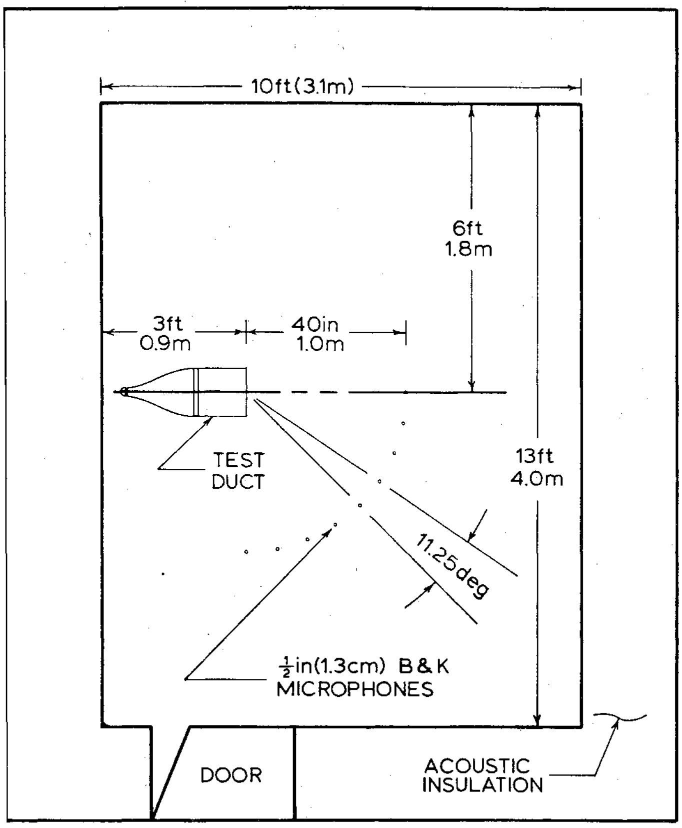

Fig. 6 Anechoic chamber (plan view) with test setup.

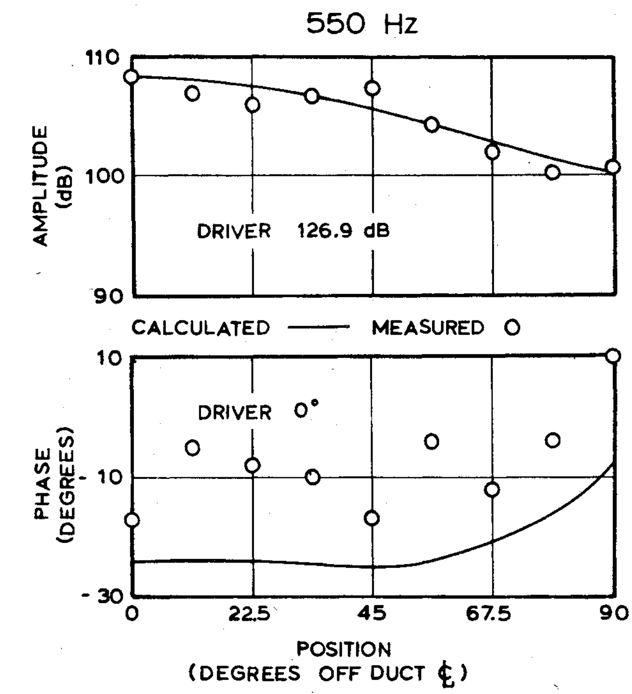

Fig. 7 Acoustic radiation 101.6 cm (40 in.) from hard-walled straight duct.

assuming plane wave excitation at the driver plane. Since a plane wave was assumed, the phase was not measured at the driver plane and zero phase was assumed for this location in the theoretical model. This can be done without any loss of generality as the phase differences between the field microphones and the driver plane microphone are the quantities of interest which were measured in this program. Once the driver plane acoustic pressure was input into the theoretical model, the far-field sound distribution was calculated and compared with the measured experimental data.

The computer used for these analyses was the Georgia Tech CDC Cyber 70/74. Typical run times to calculate the surface distribution of the acoustic potential with 100 points on the theoretical body are about 6 min and to calculate the acoustic potential at 20 points in the far field are about 1 min. In the theoretical calculations of the surface potential on the straight duct (see Fig. 1) 102 points were used, while in the calculations for the inlet (see Fig. 4) 97 points were used.

Results

Tests with the straight duct configuration with both hard and soft walls were conducted in the frequency range of 300-700 Hz at 50 Hz increments. The lower frequency limit was imposed by the limitations of the University acoustic driver employed, and the upper limit represented cut-on of the $1T$ mode. Over this frequency range a reasonably plane wave could be excited at the driver plane. Comparisons of calculated and measured data for the hard-walled straight duct configuration are presented in Fig. 7 for 550 Hz. The SPL results at this frequency are representative of those at all other frequencies while the phase results are the worst at this frequency. The probable reason for this is that 550 Hz is very close to the tuning frequency of the liner, which was calculated to be around 558 Hz under these test conditions and the tape used to close off the liner holes for the hard-walled tests was not 100% effective. The average absolute errors for the amplitude in decibels and for the phase in degrees are presented in Table 1 for all of the tests run.

To predict the sound field radiated by the lined, straight wall duct the admittance at the wall is required. In this case available liner theory[7] was used to predict the admittance of the Helmholtz resonator array. For the particular resonators used in this study (see Fig. 3) the resonant frequency of the array f_0 was calculated to be around 558 Hz and the specific impedance of the liner $Z=\theta-iX$ was found to be given by:

$$\theta=0.01398\sqrt{f}, \qquad X=10.186\left(\frac{f}{f_0}-\frac{f_0}{f}\right) \tag{1}$$

where f is the frequency in hertz.

Comparisons of experimental and calculated far-field pressures for the lined, straight wall duct are presented in Fig. 8 at a frequency of 500 Hz. These results are typical of those

Table 1 Average absolute errors

		Frequency, Hz								
		300	350	400	450	500	550	600	650	700
Hard-walled	dB	0.96	0.95	0.66	0.70	0.71	0.78	0.92	0.67	1.02
straight duct	deg	3.83	5.56	5.89	4.61	7.44	13.78	4.33	7.72	7.83
Soft-walled	dB	1.06	1.96	0.82	0.63	0.74	15.71	1.61	0.42	1.19
straight duct	deg	4.67	4.78	9.28	3.67	9.33	88.89	8.11	5.89	6.61
	dB	1.18	0.72	0.61	1.08	0.69	0.63	0.61	0.38	1.07
Inlet	deg	3.17	6.39	3.83	10.22	5.00	6.22	4.00	4.50	5.00

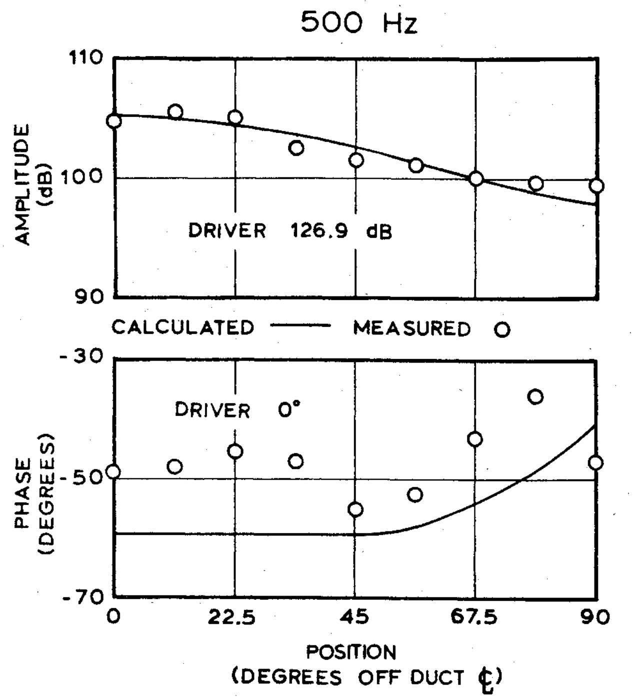

Fig. 8 Acoustic radiation 101.6 cm (40 in.) from soft-walled straight duct.

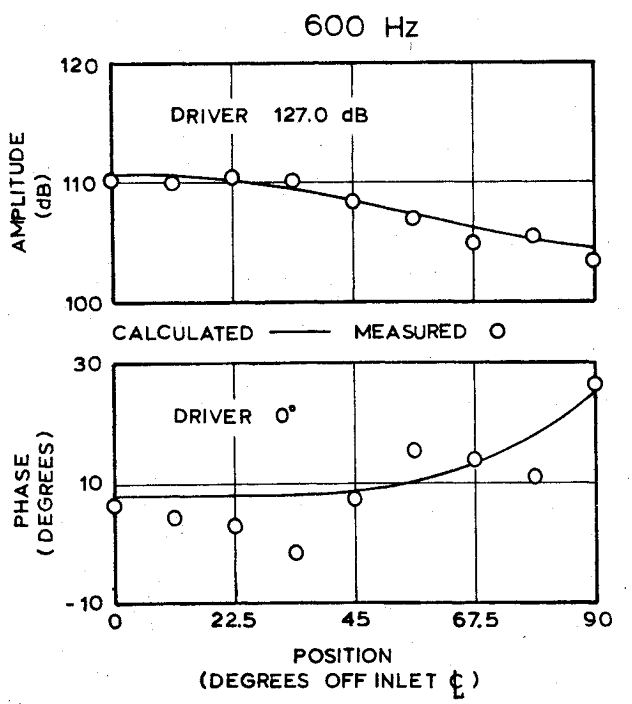

Fig. 9 Acoustic radiation 101.6 cm (40 in.) from inlet.

obtained at all but one of the other frequencies tested. At 550 Hz, which is very close to the calculated resonant frequency of the liner, the results show significantly more error (see Table 1). Since the calculated results were lower than those measured it was assumed that the liner theory results predicted the effectiveness of the liner to be higher than it actually was in practice. To see if this was actually the case, a set of systematic computer runs was made in which the effectiveness of the liner (i.e., its admittance) was reduced. It was found that if the predicted real part of the admittance was taken to be 35% effective and the imaginary part of the admittance was taken to be 65% effective (i.e., $y = -0.13 - i0.28$ at 550 Hz), the results from the theoretical calculations were significantly closer to the experimentally measured data. The average absolute errors for the amplitude in decibels and for the phase in degrees are now 2.51 dB and 5.00 deg, respectively, as compared to the much larger errors incurred using the admittance values predicted by the linear theory (see Table 1).

Tests for the inlet configuration were also conducted over the frequency range 300-700 Hz at 50 Hz increments. Since the reference lengths a are slightly different (see Figs. 1, 2, 4, and 5), the nondimensional wave numbers ka are different. Comparisons of theoretical and experimental data are presented in Fig. 9 for the inlet configuration at 600 Hz. These results are representative of those at other frequencies.

Discussion of Results

The causes for the errors appearing in the comparisons of the last section are briefly discussed herein. First, there are the obvious experimental errors caused by microphone amplifier drift, temperature changes in the anechoic chamber, and microphone placement in the chamber. These errors can be estimated, as identical tests were run on different days, the microphones were calibrated three times during the course of each test, and the microphones were moved and reset during each test as there are nine positions in the field and only five microphones were available for use in each test. Comparing test results and calibrations, these errors are estimated as being of the order of 0.5 dB in amplitude and 5 deg in phase. Another source of experimental error is the anechoic chamber itself which, as stated before, can account for up to 3 dB errors in amplitude and 10 deg errors in phase. A more subtle source of error in the lined duct tests is the liner itself. It was found that by changing the dimensions of the Helmholtz resonators by as little as 0.01 mm (0.0005 in.) (common machining errors) the resonant frequency calculated for the liner changed by as much as 1.5 Hz, which is significant for this type of liner near resonance due to its highly peaked absorption curve. A sample calculation was run at 550 Hz with this change and the calculated results changed by about 3 dB in amplitude and 5 deg in phase. Another source of possible error for this particular case is obviously the imperfection of the liner theory itself; the determination of the errors caused by its shortcomings are, however, beyond our current capabilities.

The computer programs also introduce some errors which are estimated to be about 1% by comparing these computer results with exact solutions for similar geometries and wave numbers. Although these errors are insignificant when evaluated in decibels, they can be as high as 5 deg in phase. Another source of error is the assumption of a plane wave at the driver plane; the effect of this error on the results in the far field cannot, however, be easily estimated. Other sources of error include the differences between the experimental and theoretical geometries which include not only the different terminations on the back side of the bodies but also the stand required to hold up the experimental setup in the anechoic chamber. The errors caused by these differences are hopefully small.

Conclusions

Acoustic measurements were made of the sound field radiated from a straight duct with both acoustically hard and soft walls and a jet engine inlet with plane wave excitation. These measurements were then compared with the results of a cylindrically symmetric integral representation of the solutions of the Helmholtz equation and good agreement between the theoretical and experimental results was observed. This indicates that the integral equations and the techniques employed for solving them are good approximations to the actual acoustic behavior of arbitrarily shaped axisymmetric ducts radiating into a free space. This is significant in that most theories can not adequately model the coupling between the acoustic fields inside and outside a duct. Thus, this technique can be used with confidence to efficiently predict the sound field radiated from complex axisymmetric geometries.

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References

[1] Bell, W. A., Meyer, W. L., and Zinn, B. T., "Predicting the Acoustics of Arbitrarily Shaped Bodies Using an Integral Approach," *AIAA Journal*, Vol. 15, June 1977, pp. 813-820.

[2] Meyer, W. L., Bell, W. A., Stallybrass, M. P., and Zinn, B. T., "Boundary Integral Solutions of Three Dimensional Acoustic Radiation Problems," *Journal of Sound and Vibration*, Vol. 59, July 1978, pp. 245-262.

[3] Meyer, W. L., Bell, W. A., Stallybrass, M. P., and Zinn, B. T., "Prediction of the Sound Field Radiated From Axisymmetric Surfaces," *Journal of the Acoustical Society of America*, Vol. 65, March 1979, pp. 631-638.

[4] Meyer, W. L., Bell, W. A., and Zinn, B. T., "Sound Radiation from Finite Length Axisymmetric Ducts and Engine Inlets," AIAA Paper 79-0675 presented at the AIAA 5th Aeroacoustics Conference, Seattle, Wash., March 1979.

[5] Burton, A. J., "The Solution of Helmholtz' Equation in Exterior Domains Using Integral Equations," National Physical Laboratory, Teddington, Middlesex, England, NPL Report NAC 30, Jan. 1973.

[6] Miller, B. A., Dastoli, B. J., and Wesoky, H. L., "Effect of Entry-Lip Design on Aerodynamics and Acoustics of High-Throat-Mach-Number Inlets for the Quiet, Clean, Short-Haul Experimental Engine," NASA TM X-3222, May 1975.

[7] Garrison, G. D., "A Study of the Suppression of Combustion Oscillations with Mechanical Damping Devices," Phase II Summary Report, Pratt & Whitney Aircraft Co., PWA FR-1922, July 1966.